Bachelor of Science (B.Sc.) Semester–III Examination

MATHEMATICS

(Advanced Calculus, Sequence and Series)

Paper–V

Time—Three Hours] [Maximum Marks—60

INSTRUCTIONS TO CANDIDATES

(1) Solve **ALL** the **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions No. 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) If a function f(x) is continuous in [a, b] and derivable in (a, b) then prove that there exists at least one value $c \in (a, b)$ such that :

$$\frac{f(b)-f(a)}{b-a} = f'(c).$$ 6

(B) Let $f(x,y) = \frac{y(x^2+y^2)}{y^2+(x^2+y^2)^2}$.

Show that f has limit 0 as (x, y) approaches (0, 0) on the ray $x = at$, $y = bt$, but f does not have limit zero as $(x, y) \to (0, 0)$. 6

OR

(C) By using ε–δ technique prove that the function $f(x, y) = xy$ is continuous everywhere. 6

(D) Expand $(x^2 - 2xy^2)$ in Taylor's series in powers of $(x - 1)$ and $(y + 1)$. 6

UNIT–II

2. (A) Show that the envelope of the straight line

$$y\cos\theta - x\sin\theta = a - a\sin\theta \log\tan\left(\frac{\theta}{2} + \frac{\pi}{4}\right),$$

where θ is the parameter, is the curve $y = a\cosh\left(\frac{x}{a}\right)$.

6

(B) Find the envelope of the curve $\left(x/a\right)^m + \left(y/b\right)^m = 1$

when $a^n + b^n = c^n$, where a and be are the parameters and c is a constant. 6

OR

(C) Discuss the maximum or minimum values of u given by $u = x^3y^2(1 - x - y)$. 6

(D) By using Lagrange's multiplier method, find the minimum value of $x^2 + y^2 + z^2$, given that $ax + by + cz = p$. 6

UNIT–III

3. (A) Let $<x_n>$, $<y_n>$ be two sequences such that $\lim\limits_{n \to \infty} x_n = x$ and $\lim\limits_{n \to \infty} y_n = y$, where x and y are finite numbers, then prove that $\lim\limits_{n \to \infty} (x_n + y_n) = x + y$. 6

(B) Prove that the sequence $<x_n>$, where $x_n = \frac{(2n-7)}{(3n+2)}$ is a monotonic increasing sequence. Further show that it is is bounded and bends to the limit $2/3$. 6

OR

(C) If $<x_n>$ is a sequence in R, where $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots\ldots \frac{1}{n}$, evaluate $\lim\limits_{n \to \infty} |x_{n+1} - x_n|$. Verify whether this sequence satisfies the Cauchy criterion or not ? Further show that it is monotonic increasing. 6

(D) Prove that every Cauchy sequence must be bounded. Further by using example, show that the converse does not hold. 6

UNIT–IV

4\. (A) Test for convergence of the series :

$\frac{2^p}{1^q} + \frac{3^p}{2^q} + \frac{4^p}{3^q} + \dots\dots\dots\dots,$ where p and q being positive numbers, by using comparison test. 6

(B) Prove that the series $\sum (-1)^n \frac{x^n}{n}$ converges for $|x| < 1$ and diverges for $|x| > 1$ by using ratio test. Also discuss the convergence for $|x| = 1$. 6

OR

(C) Test for convergence of the series $\sum \frac{(n+1)^n x^n}{n^{n+1}}$ by using root test and $\lim_{n \to \infty} \frac{1}{n^{1/n}} = 1$. Also discuss the convergence for $x = 1$. 6

(D) Test the convergence of an alternating series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots\dots\dots\dots\dots\dots$ Also test it for absolute convergence. 6

5

(Questions)

5. (A) Find 'c' so that $\frac{f'(c)}{F'(c)} = \frac{f(b)-f(a)}{F(b)-F(a)}$ for $f(x) = x^2$, $F(x) = x$ on [a, b]. 1½

(B) Let $f(x,y) = \frac{y^2 - x^2}{x^2 + y^2}, (x,y) \neq (0,0)$. Show that $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist by using the method of the repeated limits. 1½

(C) Find the envelope of $y = t(x - t)$, parameter being t. 1½

(D) Find the stationary points of $u = x^3 + y^3 - 3axy$. 1½

(E) Show that the sequence $<x_n>$ where $x_n = \frac{zn^2+1}{2n^2-1}, \forall n \in N$ converges to 1. 1½

(F) Show that the sequence $<x_n>$ where $x_n = n/_{n+1}, \forall n \in N$, is bounded. 1½

6

(G) Test the convergence of $\sum_{n=1}^{\infty} 1/n$ by integral test.

1½

(H) Test the alternating series $\sum \frac{(-1)^n}{2n-1}$ for convergence.

1½